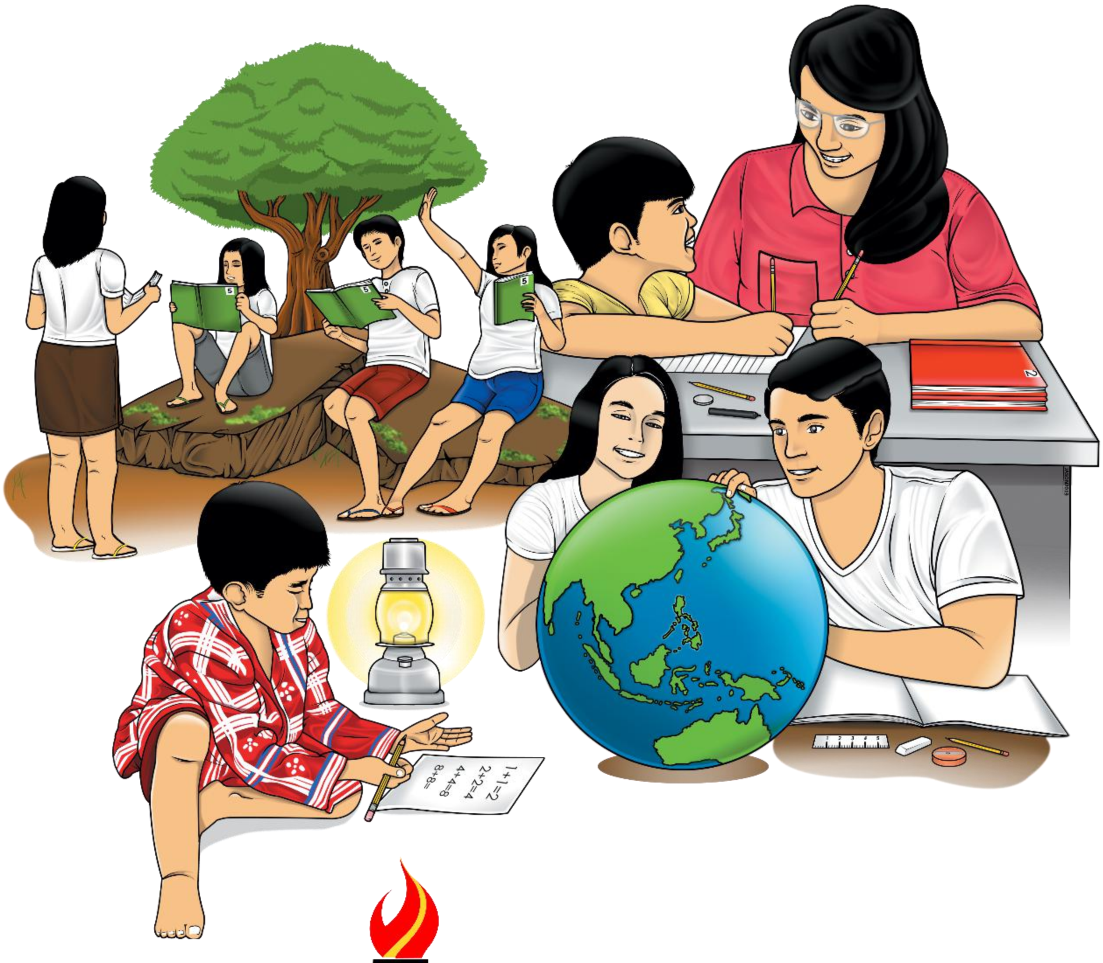
8

Mathematics

Quarter 1 – Module 5A
Multiplying and Dividing Rational Algebraic Expressions



Mathematics – Grade 8
Alternative Delivery Mode
Quarter 1 – Module 5A Solving Rational Algebraic Expressions
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Mathematics

Quarter 1 – Module 5A
Multiplying and Dividing Rational Algebraic Expressions

Introductory Message

For the facilitator:

Welcome to the Mathematics 8 Alternative Delivery Mode (ADM) Module on Multiplying and Dividing Rational Algebraic Expressions!

This module was collaboratively designed, developed and reviewed by educators both from public and private institutions to assist you, the teacher or facilitator in helping the learners meet the standards set by the K to 12 Curriculum while overcoming their personal, social, and economic constraints in schooling.

This learning resource hopes to engage the learners into guided and independent learning activities at their own pace and time. Furthermore, this also aims to help learners acquire the needed 21st century skills while taking into consideration their needs and circumstances.

As a facilitator, you are expected to orient the learners on how to use this module. You also need to keep track of the learners' progress while allowing them to manage their own learning. Furthermore, you are expected to encourage and assist the learners as they do the tasks included in the module.

For the learner:

Welcome to the Mathematics 8 Alternative Delivery Mode (ADM) Module on Multiplying and Dividing Rational Algebraic Expressions!

This module was designed to provide you with fun and meaningful opportunities for guided and independent learning at your own pace and time. You will be enabled to process the contents of the learning resource while being an active learner.

This module has the following parts and corresponding icons:

What I Need to Know	This will give you an idea of the skills or competencies you are expected to learn in the module.
What I Know	This part includes an activity that aims to check what you already know about the lesson to take. If you get all the answers correct (100%), you may decide to skip this module.
What's In	This is a brief drill or review to help you link the current lesson with the previous one.
What's New	In this portion, the new lesson will be introduced to you in various ways; a story, a song, a poem, a problem opener, an activity or a situation.
What is It	This section provides a brief discussion of the lesson. This aims to help you discover and understand new concepts and skills.
What's More	This comprises activities for independent practice to solidify your understanding and skills of the topic. You may check the answers to the exercises using the Answer Key at the end of the module.
What I Have Learned	This includes questions or blank sentence/paragraph to be filled in to process what you learned from the lesson.
What I Can Do	This section provides an activity which will help you transfer your new knowledge or skill into real life situations or concerns.
Assessment	This is a task which aims to evaluate your level of mastery in achieving the learning competency.
Additional Activities	In this portion, another activity will be given to you to enrich your knowledge or skill of the lesson learned.
Answer Key	This contains answers to all activities in the module.

At the end of this module you will also find:

References — This is a list of all sources used in developing this module.

The following are some reminders in using this module:

1. Use the module with care. Do not put unnecessary mark/s on any part of the module. Use a separate sheet of paper in answering the exercises.
2. Don’t forget to answer *What I Know* before moving on to the other activities included in the module.
3. Read the instruction carefully before doing each task.
4. Observe honesty and integrity in doing the tasks and checking your answers.
5. Finish the task at hand before proceeding to the next.
6. Return this module to your teacher/facilitator once you are through with it.

If you encounter any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator. Always bear in mind that you are not alone.

We hope that through this material, you will experience meaningful learning and gain deep understanding of the relevant competencies. You can do it!

What I Need to Know

This module covers key concepts of operations on rational algebraic expressions. You are also given varied activities to process your knowledge and skills learned to deepen and transfer your understanding of the different lessons.

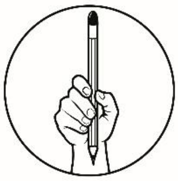
The module is divided into the following lessons:

Lesson 1: Multiplying Rational Algebraic Expressions; and

Lesson 2: Dividing Rational Algebraic Expressions.

In going through this module, you are expected to:

1. multiply rational algebraic expressions; and
2. divide rational algebraic expressions;



What I Know

Choose the correct answer. Write your answer on a separate sheet of paper. If you get a perfect score, you may skip this module.

1. Give the reduced form of $\frac{10}{2ab^2} \cdot \frac{a^2\,b^2}{5}$.
   A. a
   B. b
   C. *2*
   D. *5*
2. What are the common factors in the numerators and denominators of $\frac{6ab^5}{bc^2} \cdot \frac{ab^3c}{a^3bc}$?
   A. abc
   B. a^2bc
   C. ab^2c
   D. a^2b^2c
3. Find the product of $\frac{4}{6-2a} \cdot \frac{3-a}{2}$.
   A. 1
   B. 2
   C. 3
   D. 4
4. Write as one fraction and simplify $\frac{2}{x^2+x} \cdot \frac{2x+2}{x}$.
   A. $\frac{3}{x^2}$
   B. $\frac{4}{x^2}$
   C. $\frac{6}{x^2}$
   D. $\frac{8}{x^2}$
5. Find the common factors of the numerator and denominator of $\frac{x^2-16}{x^2+x-20} \cdot \frac{x^2-8x+16}{2x-8}$.
   A. $x-3$
   B. $x-4$
   C. $(x-3)(x-4)$
   D. $(x-4)(x-4)$
6. What is the product of $\frac{x-3}{x^2+x-20} \cdot \frac{x^2-8x+16}{x-3}$?
   A. $\frac{x-4}{x-5}$
   B. $\frac{x-4}{x+5}$
   C. $\frac{x+4}{x-5}$
   D. $\frac{x+4}{x+5}$
7. What is the quotient of $\frac{2x+5}{4} \div \frac{2x+5}{6}$?
   A. $\frac{1}{4}$
   B. $\frac{1}{2}$
   C. $\frac{2}{3}$
   D. $\frac{3}{2}$
8. From $\frac{x-5}{5} \div \frac{x-2}{3}$, what is the divisor?
   A. $\frac{x-2}{3}$
   B. $\frac{x-5}{5}$
   C. $\frac{x-3}{x-5}$
   D. $\frac{x-5}{x-3}$
9. What common factors can be found in dividing $\frac{2x-5}{4} \div \frac{4x-10}{4}$?
   A. 2 and $x-5$
   B. 4 and $x-10$
   C. 4 and $2x-5$
   D. 4 and $4x-10$

10. Perform the indicated operation $\frac{x-2}{2} \div \frac{x^2-4}{3}$.

A. $\frac{3}{2x+4}$

B. $\frac{4}{2x-4}$

C. $\frac{6}{2(x+2)}$

D. $\frac{8}{2(x-2)}$

11. Given $\frac{x-4}{2}$ as one factor of the rational algebraic expression $\frac{x^2-16}{4}$, what is the other factor?

A. $\frac{x+2}{2}$

B. $\frac{x+4}{2}$

C. $\frac{x+6}{2}$

D. $\frac{x+8}{2}$

12. Divide $\frac{r+1}{r-2}$ with $\frac{2r+2}{r+2}$.

A. $\frac{r-2}{2r}$

B. $\frac{r+2}{2r-1}$

C. $\frac{r+2}{2r-4}$

D. $\frac{r-2}{2r+4}$

13. Below are the steps in finding the quotients of rational algebraic expressions. Which of the following is in correct order?

I. Divide out common factors.
II. Simplify the remaining factors.
III. Find the reciprocal of the divisor and proceed to multiplication.
IV. Determine the dividend and divisor of the given expression.

A. I, II, III

B. II, III, IV

C. IV, III, I, II

D. IV, I, II, III

14. Given the expression $\frac{a^2+a}{3a-15} \div \frac{a^2+2a+1}{6a-30}$, which of the following statements is true?

A. The divisor is $\frac{a^2+a}{3a-15}$.

B. The reciprocal of the divisor is $\frac{3a-15}{a^2+a}$.

C. The common factors that can be divided out are $(a+1)(3a-15)$.

D. The quotient obtained after dividing the expressions $\frac{a^2+a}{3a-15} \div \frac{a^2+2a+1}{6a-30}$ is $\frac{2a}{a+1}$.

15. Your classmate multiplied rational algebraic expressions as presented below.

$$\frac{x+3}{2} \cdot \frac{2}{x+3} = \frac{x+3}{2} \cdot \frac{x+3}{2} = \frac{x^2+6x+9}{4}$$

Is your classmate's solution correct?

A. Yes. There is a need to get the reciprocal of the second factor.

B. Yes. The product obtained after multiplying the expressions is correct.

C. No. Getting the reciprocal is not applicable in multiplying rational expressions.

D. No. The product obtained after multiplying the rational algebraic expressions is wrong.

Lesson 1 Multiplying Rational Algebraic Expressions

Whether you go by the boat, by car, or by plane, traveling can be a lot of fun. It is also very educational as it gives you first hand experiences about things you just see on TV or read in the papers or in books.

But do you realize how much mathematical concepts can be involved in traveling? For example, do you know how far you can reach by a bus if it is traveling $\frac{37}{2}$ km/h for $\frac{1}{4}$ of an hour?

What's In

Rational algebraic expressions are multiplied the same way as you would multiply regular fractions. So, let us recall multiplication of fractions.

Directions: Match the expressions in Column A with its product in Column B and answer the questions that follow. Use another paper for your answer.

	Column A		Column B
1.	$\frac{4}{3}\cdot\frac{5}{7}$	A.	$\frac{1}{6}$
2.	$\frac{2}{5}\cdot\frac{3}{2}$	B.	$\frac{20}{21}$
3.	$\frac{5}{10}\cdot\frac{2}{6}$	C.	$\frac{3}{5}$
4.	$\frac{3}{9}\cdot\frac{12}{15}$	D.	$\frac{9}{5}$
5.	$\frac{3}{5}\cdot 3$	E.	$\frac{4}{15}$
		F.	$\frac{3}{4}$

Questions:

1. Do the fractions in Item 1 of Column A have greatest common factor (GCF) other than 1?
2. Do the fractions in Items 2 – 4 in Column A have GCFs other than 1? How did you find them?
3. What did you do to the GCFs of Items 2-4 in Column A? Why?
4. Based from your experience of answering the items in the activity, were you able to divide out a factor of one fraction paired with a factor of the other fraction? Was it allowed?
5. How do you multiply fractions?

What's New

Situation:

Corona Virus Disease 2019 (CoVID-19) affects almost everyone around the globe. Governments have employed various proactive measures to flatten the curve of its spread. All these measures head towards the aim of keeping everyone at home as much as possible. In effect, many have lost their jobs and food on the table become scarce. Driven by the spirit of helpfulness, your churchmates distribute relief goods to different places. Using the church's vehicle, they travel at a speed of $30\ km/hr$. How far will your churchmates reach after 25 minutes of travel? Suppose the distance (in km) is represented by x, and the time (in $hours$) is represented by y, how far will your churchmates reach if they travel $\frac{y+1}{14}\ hours$? $\frac{y-5}{20}\ hours$? Complete the table below and answer the questions that follow. The first one is done for illustration. Remember that $Speed \cdot Time = Distance$.

	Speed	Time	Distance
1	$30\ km/hr$	$25\ minutes = \dfrac{1}{4} hr.$	$\dfrac{30}{4} km = 7\dfrac{1}{2} km$
2	$30\ km/hr$	$\dfrac{y+1}{14}\ hrs.$	
3	$30\ km/hr$	$\dfrac{y-5}{20}\ hrs.$	

Questions:

1. How did you solve for distance?
2. In answering the activity, what did you do to the common factors in the numerator and denominator? Why?
3. What do you call each entry of Columns 2 and 3?
4. How do you multiply rational algebraic expressions?

What is It

The concept of multiplying the numerators and the denominators of fractions also applies to multiplying rational algebraic expressions. For example, the distance in the previous activity is solved by multiplying the numerators and denominators of the speed and time of travel. In addition to just multiplying the numerators and denominators, there are methods of reducing the product into its lowest forms. Multiplying rational algebraic expressions and methods of reducing the product in lowest form will be discussed using series of examples.

The first two examples will be using monomials to illustrate multiplication of rational algebraic expressions.

Method 1: Divide Out Greatest Common Monomial Factor (GCMF) After Multiplying

Example 1: $\frac{37x}{2y} \cdot \frac{y}{4}$

Solution:

Step 1. Multiply the numerators and denominators of the given rational algebraic expressions.

$37x \cdot y$	$= 37xy$	Multiply the numerators.
$2y \cdot 4$	$= 8y$	Multiply the denominators.

Step 2: Find the GCMF of the product of numerators and denominators.

$37xy$	$= 37 \cdot x \cdot \boldsymbol{y}$	Look for prime factors.
$8y$	$= 2 \cdot 2 \cdot 2 \cdot \boldsymbol{y}$	Look for prime factors.
GCF	$= y$	Look for the common factors from the two groups of factors.

Step 3: Divide out the GCMF of the product of numerators and denominators.

$\frac{37x}{2y} \cdot \frac{y}{4}$	$= \frac{37xy}{8y}$	Product (not yet reduced)
	$= \frac{37x\cancel{\boldsymbol{y}}}{8\cancel{\boldsymbol{y}}}$	Divide out GCMF.

Step 4: Simplify the remaining factors.

$\frac{37x}{2y} \cdot \frac{y}{4}$	$= \boldsymbol{\frac{37x}{8}}$	**Product in reduced form.**

Method 2: Divide Out Greatest Common Monomial Factor (GCMF) Before Multiplying

Example 2: $\frac{15y}{2y} \cdot \frac{2y}{5x^2}$

Solution:

Step 1. Find the GCMF of the of numerators and denominators

$15y \cdot 2y$	$= 3 \cdot \boldsymbol{5} \cdot \boldsymbol{y} \cdot \boldsymbol{2} \cdot y$	Look for prime factors of the two numerators.
$2y \cdot 5x^2$	$= \boldsymbol{2} \cdot \boldsymbol{y} \cdot \boldsymbol{5} \cdot x \cdot x$	Look for prime factors of the two denominators.
GCF	$= \boldsymbol{2} \cdot \boldsymbol{y} \cdot \boldsymbol{5}$	Look for common factors from the numerators and

denominators.

Step 2: Divide out the GCMF of the numerators and denominators.

$$\frac{15y}{2y}\cdot\frac{2y}{5x^2} = \frac{3\cdot 5\cdot y}{2\cdot y}\cdot\frac{2\cdot y}{5\cdot x\cdot x}$$ Factorization of the numerators and denominators.

$$= \frac{3\cdot\cancel{\mathbf{5}}\cdot\cancel{\boldsymbol{y}}}{\cancel{\mathbf{2}}\cancel{\boldsymbol{y}}}\cdot\frac{\cancel{\mathbf{2}}\cdot y}{\cancel{\mathbf{5}}\cdot x\cdot x}$$ Divide out GCMF.

Step 3: Simplify the remaining factors.

$$\frac{15y}{2y}\cdot\frac{2y}{5x^2} = \frac{3}{1}\cdot\frac{y}{x\cdot x}$$ Remaining factors

$$= \frac{3}{1}\cdot\frac{y}{x\cdot x}$$ Multiply the numerators. Multiply the denominators.

$$= \frac{\mathbf{3}\boldsymbol{y}}{\boldsymbol{x}^{\mathbf{2}}}$$ **Product in reduced form.**

The methods illustrated by the previous examples can also be used to rational algebraic expressions involving polynomials.

Method 1: Divide Out Greatest Common Monomial Factor (GCMF) After Multiplying

Example 1: $\frac{x^2-4}{2}\cdot\frac{4}{x-2}$

Solution:

Step 1. Multiply the numerators and denominators of the given rational algebraic expressions.

$$(x^2-4)(4) = (x^2)(4)-(4)(4)$$ Distributive Property

$$= 4x^2-16$$ Simplified

$$(2)(x-2) = (2)(x)-(2)(2)$$ Distributive Property

$$= 2x-4$$ Simplified

Step 2: Find the GCMF of the product of numerators and denominators.

$$4x^2-16 = (2x-4)(2x+4)$$ Factoring Difference of Two Squares

$$= (\mathbf{2})(\boldsymbol{x}-\mathbf{2})(2)(x+2)$$ Factoring the Greatest Common Monomial Factor (GCMF)

$$2x-4 = \mathbf{2}(\boldsymbol{x}-\mathbf{2})$$ Factoring the Greatest Common Monomial

			Factor (GCMF)
$GCMF$	$=$	$\mathbf{2(x-2)}$	Look for the common factors from the numerators and denominators.

Step 3: Divide out the GCMF of the product of numerators and denominators.

$\frac{x^2-4}{2}\cdot\frac{4}{x-2}$	$=$	$\frac{4x^2-16}{2x-4}$	Product (not yet reduced)
	$=$	$\frac{(2)(x-2)(2)(x+2)}{2(x-2)}$	Factored form of the numerator and denominator.
	$=$	$\frac{\cancel{(2)}\cancel{(x-2)}(2)(x+2)}{\cancel{2}\cancel{(x-2)}}$	Divide out the GCMF.

Step 4: Simplify the remaining factors.

$\frac{x^2-4}{2}\cdot\frac{4}{x-2}$	$=$	$2(x+2)$	Remaining factors
	$=$	$(2)(x)+(2)(2)$	Distributive Property
	$=$	$\mathbf{2x+4}$	**Product in reduced form.**

Method 2: Divide Out Greatest Common Monomial Factor (GCMF) Before Multiplying

Example 2: $\frac{x-5}{x^2-7x+10}\cdot\frac{x^2+x-6}{5}$

Solution:

Step 1. Find the GCMF of the of numerators and denominators

$(x-5)(x^2+x-6)$	$=$	$(x-5)\underbrace{(x^2+x-6)}$	Factor the trinomial.
	$=$	$(\mathbf{x-5})(x+3)(\mathbf{x-2})$	Retain the other factor and write the factors of the trinomial.
$(x^2-7x+10)(5)$	$=$	$\underbrace{(x^2-7x+10)}(5)$	Factor the trinomial.
	$=$	$(\mathbf{x-5})(\mathbf{x-2})(5)$	Write the factors of the trinomial and retain the other factor.
GCF	$=$	$(\mathbf{x-5})(\mathbf{x-2})$	Look for common factors from the numerators and denominators.

Step 2: Divide out the GCMF of the numerators and denominators.

$$\frac{x-5}{x^2-7x+10}\cdot\frac{x^2+x-6}{5} \quad = \quad \frac{(x-5)(x+3)(x-2)}{(x-5)(x-2)(5)}$$ Factorization of the numerators and denominators.

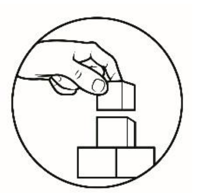
$$= \quad \frac{\cancel{(\boldsymbol{x-5})}(x+3)\cancel{(\boldsymbol{x-2})}}{\cancel{(\boldsymbol{x-5})}\cancel{(\boldsymbol{x-2})}(5)}$$ Divide out GCMF.

Step 3: Simplify the remaining factors.

$$\frac{x-5}{x^2-7x+10}\cdot\frac{x^2+x-6}{5} \quad = \quad \frac{\boldsymbol{x+3}}{\boldsymbol{5}}$$ **Product in reduced form.**

DON'T FORGET

In multiplying rational algebraic expressions, either before or after multiplying, always divide out the common factors to attain the product in reduced form.



What's More

A. Find the product of the following rational algebraic expressions by **dividing out the GCMF after multiplying**. Use another paper for your answers.

1. $\frac{3x}{4}\cdot\frac{8}{9}$ 2. $\frac{2x-2}{3}\cdot\frac{2}{x^2-1}$

Questions:

1. What was the first step that you did to find the products of the problems given?
2. What did you do to the GCMFs that you found in each of the items? Why?
3. What did you do last?

B. Find the product of the following rational algebraic expressions by **dividing out the GCMF before multiplying**.

1. $\frac{7}{2x^3}\cdot\frac{x^2}{21}$ 2. $\frac{x+2}{x^2-14x+49}\cdot\frac{x-7}{2}$

Questions:

1. What was the first step that you did to find the products of the problems given?
2. What did you do to the GCMFs that you found in each of the items? Why?

3. What did you do last?

C. Find the product. You may use any of the discussed methods to perform the operation.

1. $\dfrac{7}{x^2-4} \cdot \dfrac{x(x-2)}{14}$

Questions:

1. What method did you use?
2. What made you decide whether to use Method 1 or Method 2?

What I Have Learned

Situation: Your classmate asks for help to complete the solution-explanation card of the problem below. Please do help!

$$(3y) \cdot \frac{y^2+3y-4}{y^2+5y+4}$$

Solution

$$(3y) \cdot \frac{y^2+3y-4}{y^2+5y+4} = (\_\_\_\_) \cdot \frac{(y+4)(\_\_\_\_\_)}{(y+1)(\_\_\_\_\_)}$$

$$= (\_\_\_\_) \cdot \frac{(y+4)(\_\_\_\_\_)}{(y+1)(\_\_\_\_\_)}$$

$$= (\_\_\_\_\_\_)(\_\_\_\_\_\_)$$

$$= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

Explanation

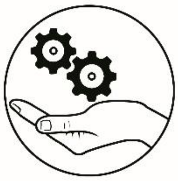
I know how to ________________.

First, ________________________.

After that, ____________________.

Then, ________________________.

Finally, ______________________.



What I Can Do

Situation: Your friend is planting sweet potato on a rectangular plot. Suppose the number of sweet potatoes planted is represented by x and the length of the plot is $\frac{x-3}{4}$ *units* while the width is $\frac{2x}{x+3}$ *units*.

Questions:
1. What shall your friend do to find the area of the rectangular plot?
2. What is the area of the rectangular plot? Show your solution.

Lesson 2 Dividing Rational Algebraic Expressions

The previous lesson made you understand that multiplying rational algebraic expressions is the same with multiplying fractions. Do you think dividing rational algebraic expressions is also the same with dividing fractions?

What's In

Recall dividing fractions. Match the expression in Column A with its quotient in Column B. Write your answers on another sheet of paper.

Column A		Column B	
1.	$\frac{4}{3} \div \frac{5}{7}$	A.	$\frac{3}{5}$
2.	$\frac{2}{5} \div \frac{2}{3}$	B.	2
3.	$6 \div \frac{6}{2}$	C.	$\frac{3}{28}$
4.	$\frac{3}{7} \div 4$	D.	1
5.	$\frac{3}{5} \div \frac{3}{5}$	E.	$\frac{4}{15}$
		F.	$\frac{28}{15}$

Questions:

1. In each item in Column A, which of the expression is what you call the divisor – is it the fraction that comes first or second?
2. What did you do to the divisors to find the quotients?
3. What operation did you use to replace the division operation?
4. What did you do to the Greatest Common Factor (GCF) of Items 2, 3, and 5? Why did you do so?
5. How do you divide fractions?

What's New

Ellen, a Grade 8 student, was assigned to solve the problem: $\frac{x^2-9}{x-3} \div \frac{x+3}{x-3}$. To solve this, she performed the following steps:

Step 1. Applying her knowledge in dividing fractions, she rewrote the problem as

$$\frac{x^2-9}{x-3} \cdot \frac{x-3}{x+3}$$

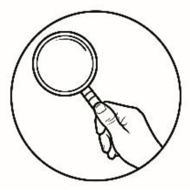
Step 2. Multiply.

$$\frac{x^2-9}{x-3} \cdot \frac{x-3}{x+3} \quad = \quad \frac{x^3-9x-3x^2+27}{x^2-6x+9}$$

Step 3. Ellen concluded that $\frac{x^3-9x-3x^2+27}{x^2-6x+9}$ is the quotient of the given rational algebraic expressions in its simplest form.

Question:

1. Is Ellen's answer correct? Elaborate your answer.
2. What did Ellen do in Step 1?
3. What suggestions could you make to help Ellen in Step 2?
4. What is the quotient of the given rational algebraic expressions?
5. How would you divide the rational algebraic expressions?



What is It

The quotient of two rational algebraic expressions is the product of the dividend and the reciprocal of the divisor. In symbols,

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}, b, c, d \neq 0$$

To help you understand how to divide rational algebraic expressions, examine the examples below.

Example 1: $\frac{x+y}{x-y} \div \frac{x}{y}$

Solution:

Step 1. Determine the dividend and divisor of the expression.

$$\underset{\Downarrow}{\frac{x+y}{x-y}} \quad \div \quad \underset{\Downarrow}{\frac{x}{y}}$$

$$Dividend \qquad Divisor$$

Step 2: Find the reciprocal of the divisor.

$$\frac{x}{y} \Rightarrow \frac{y}{x}$$

Reciprocal means its multiplicative inverse.

Step 3: Multiply the dividend with the reciprocal of the divisor.

$$\frac{x+y}{x-y} \div \frac{x}{y} = \frac{(x+y)}{(x-y)} \cdot \frac{y}{x}$$

Multiply numerator with numerator.

$$= \frac{(x+y)}{(x-y)} \cdot \frac{y}{x}$$

Multiply denominator with denominator.

$$= \frac{(x)(y) + (y)(y)}{(x)(x) - (y)(x)}$$

Distributive Property

$$= \frac{xy + y^2}{x^2 - xy}$$

Quotient in reduced form

Example 2: $\frac{3p^2+6p+3}{p+1} \div \frac{3}{p}$

Solution:

Step 1. Determine the dividend and divisor of the expression.

$$\underbrace{\frac{3p^2+6p+3}{p+1}}_{Dividend} \div \underbrace{\frac{3}{p}}_{Divisor}$$

Step 2: Find the reciprocal of the divisor.

$$\frac{3}{p} \Rightarrow \frac{p}{3}$$

Reciprocal of the divisor is its multiplicative inverse.

Step 3: Multiply the dividend with the reciprocal of the divisor.

$\frac{3p^2+6p+3}{p+1} \div \frac{3}{p} = \frac{3p^2+6p+3}{p+1} \cdot \frac{p}{3}$	Dividend times the reciprocal of divisor.
$= \frac{(3p+3)(p+1)}{(p+1)} \cdot \frac{(p)}{(3)}$	Factoring Trinomial (numerator 1)
$= \frac{(3)(p+1)(p+1)}{(p+1)} \cdot \frac{(p)}{(3)}$	Factoring the GCMF (numerator 1)
$= \frac{\cancel{(3)}\cancel{(p+1)}(p+1)}{\cancel{(p+1)}} \cdot \frac{(p)}{\cancel{(3)}}$	Divide out GCMF.
$= (p+1)p$	Simplify the remaining factors.
$= (p)(p) + (1)(p)$	Distributive Property
$= p^2+1$	Quotient in reduced form

Example 3: $\frac{x^2+5x+6}{x^2+4x+4} \div \frac{x+1}{x+3}$

Solution:

Step 1. Determine the dividend and divisor of the expression.

$$\underbrace{\frac{x^2+5x+6}{x^2+4x+4}}_{Dividend} \div \underbrace{\frac{x+1}{x+3}}_{Divisor}$$

Step 2: Find the reciprocal of the divisor.

$$\frac{x+1}{x+3} \Rightarrow \frac{x+3}{x+1}$$

Reciprocal of the divisor is its multiplicative inverse.

Step 3: Multiply the dividend with the reciprocal of the divisor.

$\dfrac{x^2+5x+6}{x^2+4x+4} \div \dfrac{x+1}{x+3}$	$= \dfrac{x^2+5x+6}{x^2+4x+4} \cdot \dfrac{x+3}{x+1}$	Dividend times the reciprocal of the divisor.
	$= \dfrac{(x+2)(x+3)}{(x+2)(x+2)} \cdot \dfrac{(x+3)}{(x+1)}$	Factoring Trinomial (numerator 1 and denominator 1)
	$= \dfrac{\cancel{(x+2)}(x+3)}{\cancel{(x+2)}(x+2)} \cdot \dfrac{(x+3)}{(x+1)}$	Divide out GCMF.
	$= \dfrac{(x+3)\,(x+3)}{(x+2)\,(x+1)}$	Use FOIL to simplify the remaining factors. (See process below.)
	$= \dfrac{x^2+6x+9}{x^2+3x+2}$	Quotient

F **L**

$$(x+3)(x+3)$$

I

O

Terms Multiplied	Expression	Product	Final Product
First Terms	$(x)(x)$	$= x^2$	$= x^2$
Outer Terms	$(x)(3)$	$= 3x$	$= 6x$
Inner Terms	$(3)(x)$	$= 3x$	
Last Terms	$(3)(3)$	$= 9$	$= 9$
			$\boldsymbol{= x^2 + 6x + 9}$

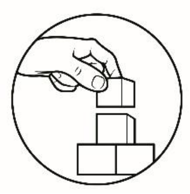
F **L**

$$(x+2)(x+1)$$

I

O

Terms Multiplied	Expression	Product	Final Product
First Terms	$(x)(x)$	$= x^2$	$= x^2$
Outer Terms	$(x)(1)$	$= x$	$= 3x$
Inner Terms	$(2)(x)$	$= 2x$	
Last Terms	$(2)(1)$	$= 2$	$= 2$
			$\boldsymbol{= x^2 + 3x + 2}$



What's More

Divide the following rational algebraic expressions and answer the questions that follow.

A. $\frac{a-b}{a+b} \div \frac{b}{a}$

B. $\frac{2x^2+4x+2}{x+1} \div \frac{2}{x}$

Questions:

1. From the expressions, which is the divisor? What did you do to it to get the quotient?
2. What factoring techniques did you use to find the GCMF? What did you do to the GCMFs?
3. How did you simplify the quotient?

C. $\frac{x^2+6x+9}{x^2+3x+2} \div \frac{x+3}{x-1}$

Questions:

1. From the expression, which is the divisor? What did you do to the divisor to find the quotient?
2. What factoring techniques did you use to find the GCMF? What did you do to the GCMFs?
3. How did you simplify the resulting expression?

What I Have Learned

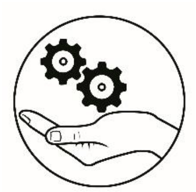
Situation: You take the lead in explaining to your group the solution of the problem below. Complete your explanation below. You may choose words, terms, or phrases form the box.

$$\frac{x^2-5x+6}{y^2} \div \frac{x-2}{y}$$

As you all know, the problem is telling us to ____. To begin solving this, we have to ____. You have to remember that the _____________________ is the expression that follows after the ________________________. So now, because the divisor is _________________________, its reciprocal is ____________________________. Then, we need to factor our numerators and __________________. By Factoring Trinomial, $x^2 - 5x +$

$6 = (\_\_\_\_\_\_)(\_\_\_\_\_\_)$. After factoring we have to change the operation to be used, from division it will become ___________. Then, we have to ___________________ common factors to ensure that our quotient will be in __________________. So, the common factors to be divided out are ______________________. Finally, the simplified remaining factors ________________________ is our quotient.

determine the divisor	divisor	multiplication	$(x-2)$	denominator
divide out	$\frac{x-3}{y}$	division sign	$\frac{x-2}{y}$	y
reduced form	divide rational algebraic expressions		$(x-3)(x-2)$	$\frac{y}{x-2}$



What I Can Do

Read the situation below and answer the questions that follow.

Your school organized a tree planting activity participated by all learners and teachers in Grades 7 to 10. To protect the newly planted trees from the harsh environment, a triangular tree guard was installed. The base (b) of one side of the triangular tree guard in terms of x is $\frac{2x^2+4x+2}{x-1}$ $units$ and its area (A) is x^2+5x+4 square units.

Questions:

1. What is the height (h) of the triangular tree guard in terms of x ? (Recall that $A = \frac{1}{2}bh$).

2. What did you do to find the height of the triangular tree guard?

Assessment

Choose the correct answer. Write your answers on a separate sheet of paper.

1. What is the reduced form of $\frac{4}{2ab^2} \cdot \frac{ab}{4}$?

   A. $\frac{1}{5b}$
   B. $\frac{1}{4b}$
   C. $\frac{1}{3b}$
   D. $\frac{1}{2b}$

2. What are the common factors in the numerators and denominators of $\frac{2a}{bc} \cdot \frac{abc}{a}$?

A. abc
B. a^2bc
C. a^2bc
D. abc^2

3. Find the product of $\frac{7}{8-2a} \cdot \frac{4-a}{2}$.

A. $\frac{5}{4}$
B. $\frac{6}{4}$
C. $\frac{7}{4}$
D. $\frac{8}{4}$

4. What is the simplest form of $\frac{2}{x^2+x} \cdot \frac{3x+3}{x}$?

A. $\frac{6}{x^5}$
B. $\frac{6}{x^4}$
C. $\frac{6}{x^3}$
D. $\frac{6}{x^2}$

5. What are the common factors in the numerators and denominators of $\frac{x^2-9}{x^2+x-20} \cdot \frac{x^2-8x+16}{3x-9}$?

A. $x+3$
B. $x-4$
C. $(x+3)(x-4)$
D. $(x-3)(x-4)$

6. Find the product of $\frac{x^2-9}{x^2+x-20} \cdot \frac{x^2-8x+16}{3x-9}$.

A. $\frac{x^2-x-10}{3x+12}$
B. $\frac{x^2-x-12}{3x+15}$
C. $\frac{x^2-x-14}{3x+18}$
D. $\frac{x^2-x-16}{3x+21}$

7. What is the quotient of $\frac{x-2}{2} \div \frac{x-2}{5}$?

A. $\frac{5}{2}$
B. $\frac{2}{5}$
C. $\frac{x-2}{5}$
D. $\frac{x-2}{10}$

8. Given $\frac{x+3}{3}$ as one factor of the rational algebraic expression $\frac{x^2-9}{6}$, what is the other factor?

A. $\frac{x-2}{1}$
B. $\frac{x-3}{2}$
C. $\frac{x-4}{3}$
D. $\frac{x-5}{4}$

9. Find the quotient of $\frac{2x-5}{4} \div \frac{x+3}{4}$.

A. $\frac{2x-5}{x+3}$
B. $\frac{4x+12}{8x-20}$
C. $\frac{8x-20}{4x+12}$
D. $\frac{2x^2+x-15}{16}$

10. What is the divisor in the given expression $\frac{2x-5}{4} \div \frac{x+3}{4}$?

A. $\frac{x+3}{4}$
B. $\frac{2x-5}{4}$
C. $\frac{x+3}{2x-5}$
D. $\frac{2x-5}{x+3}$

11. What factor or factors can be divided out in $\frac{2x-5}{4} \div \frac{x+3}{4}$?

A. $(x+3)(2x-5)$
B. $x+3$
C. $2x-5$
D. 4

12. What is the quotient when $\frac{r+9}{r-2}$ is divided by $\frac{2r+1}{r-2}$?

A. $\frac{r-2}{2r}$
B. $\frac{r+2}{2r-1}$
C. $\frac{r-9}{2r}$
D. $\frac{r+9}{2r+1}$

13. Below are the steps in finding the product of rational algebraic expressions. Which is the correct order?

I. Divide out the GCMF of the numerators and denominators.
II. Find the GCMF of the of numerators and denominators.
III. Simplify the remaining factors.

A. I, II, III
B. I, III, II
C. II, III, I
D. II, I, III

14. Given the expression $\frac{a^2+a}{3a-15} \div \frac{a^2+2a+1}{6a-30}$, which of the following statements is FALSE?

A. The divisor is $\frac{a^2+2a+1}{6a-30}$.
B. The reciprocal of the divisor is $\frac{6a-30}{a^2+2a+1}$.
C. The common factors that can be divided out are $3\ (a+1)(a-5)$.
D. The quotient obtained after dividing the expressions $\frac{a^2+a}{3a-15} \div \frac{a^2+2a+1}{6a-30}$ is $\frac{2a+1}{a+1}$.

15. Your classmate divided rational algebraic expressions as presented below.

$$\frac{x+3}{2} \div \frac{2}{x+3} = \frac{\cancel{x+3}}{\cancel{2}} \div \frac{\cancel{2}}{\cancel{x+3}} = 1$$

Is your classmate's solution correct?

A. Yes. The solution presented is correct.
B. Yes. Common factors can be directly divided out.
C. No. The correct answer should be equal to $\frac{4}{(x+3)^2}$.
D. No. Get the reciprocal of the divisor first, then multiply.

Additional Activities

Write a poem about multiplying and dividing rational algebraic expressions. You may include your experience in going through the activities in this lesson. Use a separate sheet of paper. The following will be the rubric for rating your output.

Categories and Criteria	Beginning (2)	Developing (3)	Accomplished (4)	Exemplary (5)
Content	Demonstrate 0 – 5 correct ideas about the lesson.	Demonstrate 6 - 7 correct ideas about the lesson.	Demonstrate 8 - 9 correct ideas about the lesson.	Demonstrate 10 or more correct ideas about the lesson.
Conventions	The poem has 51% or more errors in spelling and grammar.	The poem has 31% - 50% errors in spelling and grammar.	The poem has 11% -30% errors in spelling and grammar.	No errors in spelling and grammar.
Originality	The poem is 51% - 100 % copied from another source.	The poem 31% - 50% copied from another source.	The poem is 11% -30% copied from another source.	The poem is 0%-10% copied.

Answer Key

What I Know

1. A
2. D
3. A
4. B
5. D
6. B
7. D
8. A
9. C
10. A
11. B
12. C
13. C
14. D
15. C

What's In

1. B
2. C
3. A
4. E
5. D

Questions:

1. None.
2. Yes; By factorization.
3. Divide out common factors; To reduce the product into its lowest term.
4. Yes; Yes.
5. Multiply numerator with numerator and denominator with denominator; divide out common factors, simplify the remaining factors

What's New

2. $\frac{15(y+1)}{7} km$

3. $\frac{3(y-5)}{2} km$

Questions:

1. Multiply speed with time.

2. Common factors are divided out; To reduce the product into its lowest term.

3. Rational algebraic expressions.

4. Multiply numerator with numerator and denominator with denominator; divide out common factors, simplify the remaining factors.

What's More

A. 1. $\frac{2x}{3}$ 2. $\frac{4}{3x+3}$

Questions:

1. Multiply numerator with numerator and denominator with denominator.

2. Divide out common factors.

3. Simplify the remaining factors.

B. 1. $\frac{1}{6x}$ 2. $\frac{x+2}{2x-14}$

Questions:

1. Factorize the numerators and denominators.

2. Divide out; To reduce the product into its lowest term.

3. Simplify the remaining factors.

C. 1. $\frac{x}{2x+4}$

What I Have Learned

$$(3y)\cdot\frac{y^2+3y-4}{y^2+5y+4} = 3y.\frac{(y+4)(y-1)}{(y+1)(y+4)}$$
$$= (3y)\cdot\frac{\cancel{(y+4)}(y-1)}{(y+1)\cancel{(y+4)}}$$
$$= (3y)\cdot\frac{y-1)}{(y+1)}$$
$$= \frac{3y^2-3y}{y+1}$$

What I Can Do

1. Multiply the length and the width.

2. $A = lw$

$\frac{x-3}{4}\, units \cdot \frac{2x}{x+3}$ units = $\frac{x^2-3x}{2x+6}$ square units

Lesson 2

What's In

6. F
7. A
8. B
9. C
10. D

Questions:

6. Second.
7. Get the reciprocal or multiplicative inverse.
8. Multiplication.
9. Divide out; To reduce quotient into its lower terms.
10. Identify the dividend and divisor; determine the reciprocal of the divisor and change the operation to multiplication; factor the numerators and denominators; divide out common factors; simplify remaining factors.

What's New

1. No. $x^2 - 9$ is factorable as $(x + 3)(x - 3)$. Hence, common factor can be divided out.

2. Get the reciprocal of the divisor and change the operation to division.

3. Factor out $x^2 - 9$ first, divide out common factors before multiplying the remaining factors.

4. $x - 3$

5. Identify the dividend and divisor; determine the reciprocal of the divisor and change the operation to multiplication; factor the numerators and denominators; divide out

What's More

A. $\frac{a^2-ab}{ab+b^2}$

B. $x^2 + x$

Questions:

1. A. $\frac{b}{a}$; B. $\frac{2}{x}$; Find the reciprocal of the divisor.

2. Factoring GCMF and Factoring trinomial

3. Multiply the remaining factors by Distributive Property.

C. $\frac{x^2+2x-3}{x^2+3x+2}$

Questions:

1. $\frac{x+3}{x-1}$, Find the reciprocal of the divisor.

2. Factoring trinomial; Divide out

3. Apply Distributive Property/Multiply using the FOIL Method

What I Have Learned

As you all know, the problem is telling us to **divide rational algebraic expressions**. To begin solving this, we have to **determine the divisor**. You have to remember that the **divisor** is the expression that follows after the **division sign**. So now, because the divisor is $\frac{x-2}{y}$, its reciprocal is $\frac{y}{x-2}$. Then, we need to factor our numerators and **denominator**. By Factoring Trinomial, $x^2 - 5x + 6 = (x-3)(x-2)$. After factoring we have to change the operation to be used, from division it will become **multiplication**. Then, we have to **divide out** common factors to ensure that our quotient will be in **reduced form**. So, the common factors to be divided out are $(x-2)$ and y . Finally, the simplified remaining factors $\frac{x-3}{v}$ is our quotient.

What I Can Do

1. $A = \frac{1}{2}bh \rightarrow h = \frac{2A}{b} \rightarrow h = 2(x^2 + 5x + 4) \div \frac{2x^2+4x+2}{x-1}$

$\rightarrow 2(x^2 + 5x + 4) \cdot \frac{x-1}{2x^2+4x+2} \rightarrow \frac{2(x^2+5x+4)(x-1)}{2x^2+4x+2}$

$\rightarrow \frac{2(x+4)(x+1)(x-1)}{2(x^2+2x+1)}$

$\rightarrow \frac{2(x+4)(x+1)(x-1)}{2(x+1)(x+1)}$

$\rightarrow \frac{(x+4)(x-1)}{x+1} \rightarrow \frac{x^2+3x-4}{x+1}$

Hence, the height of the triangular tree guard is $\frac{x^2+3x-4}{x+1}$ $units.$

2. By deriving the formula for finding the height of the triangle, substitute values and perform operations on rational algebraic expressions.

Assessment

1. D
2. A
3. C
4. D
5. D
6. B
7. A
8. B
9. A
10. A
11. D
12. D
13. D
14. D
15. D

References

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